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Preliminary Exam in Complex Analysis

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Notation: $\mathbf{C}$ denotes the complex plane; $\mathbf{R}^-$ denotes the negative real axis (including the origin); and $\mathbf{D}$ denotes the unit disk.

1. Suppose $f(z)$ is analytic in $\mathbf{C} - \{0\}$ and satisfies

$$|f(z)| \leq \frac{1}{\sqrt{|z|}}, \qquad z \in \mathbf{C} - \{0\}.$$

   Show that f is identically zero.

2. Let $\Omega = \mathbf{C} - \mathbf{R}^-$.
   1) Define the principal branch of the logarithm, $Log(z)$, in the region Ω.
   2) Show that $Log(zw) = Log(z) + Log(w)$ for all z and w in the (open) right half-plane.
   3) Show that the identity in 2) does not hold for all z and w in Ω.

3. For each of the following functions find the radius of convergence for its Taylor series at the specified point.
   1) $f(z) = (\cos z)/(3z + 4)$ at $z_0 = 10$.
   2) $g(z) = 1/(z^2 + z + 1)$ at $z_0 = 0$.

4. Suppose $f(z)$ and $\overline{f(z)}$ are both analytic in $\mathbf{D}$. Show that $f(z)$ is constant.

5. Evaluate the following integrals.
   1) $\displaystyle\int_C \left(\sinh z + \frac{z}{2z+1}\right) dz$, where C is the unit circle traversed once clockwise.
   2) $\displaystyle\int_\Gamma (z+3\bar{z})\, dz$, where Γ is the path from -1 to 1 along the upper semi-circle $|z| = 1$.
   3) $\displaystyle\int_0^\pi \frac{d\theta}{2 + \cos\theta}$.