

**A CORRECTION ON  
 “SOME NONDIFFEOMORPHIC HOMEOMORPHIC  
 HOMOGENEOUS 7-MANIFOLDS WITH POSITIVE  
 SECTIONAL CURVATURE”**

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As was pointed out to us by L. Astey, E. Micha & G. Pastor the homeomorphism result in [3] Theorem 3.1 is not correct. They have examples [1] of two non-homeomorphic smooth spin manifolds as in Theorem 3.1 with fourth cohomology group of the same order where the invariants  $\bar{s}_i$  for  $1 \leq i \leq 3$  agree. In these examples the invariant  $s_2$  is different suggesting that the statement has to be modified by requesting instead of the equality of  $\bar{s}_2$  the equality of  $s_2$  (recall that  $\bar{s}_2$  was simply defined as  $2s_2$ ). Theorem 3.1 is based on Proposition 3.2 which in [2] was originally only proved in the smooth category. We assumed that the proof also works in the topological category; this is not true and Proposition 3.2 only holds in the smooth category. At the moment we do not have a classification for topological manifolds. As we will explain below, at least for smooth manifolds the invariant  $s_2$  is a homeomorphism invariant and one can obtain a homeomorphism classification of smooth manifolds from their diffeomorphism classification. A correct formulation of Theorem 3.1 is:

**Theorem 1.** *Let  $M$  and  $M'$  be smooth manifolds of type (2.1) such that  $|H^4(M; \mathbb{Z})| = |H^4(M'; \mathbb{Z})|$  which are both spin or both nonspin. Then  $M$  is diffeomorphic (homeomorphic) to  $M'$  if and only if  $s_i(M) = s_i(M')$  for  $i = 1, 2, 3$  (resp.  $28s_1(M) = 28s_1(M')$  and  $s_i(M) = s_i(M')$  for  $i = 2, 3$ ).*

Note that the applications to the homeomorphism and diffeomorphism classification of the Wallach spaces is not affected by the mistake

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since by Lemmas 4.4 and 5.1 the condition  $\bar{s}_i(M) = \bar{s}_i(M')$  for  $i = 1, 2, 3$  is equivalent to the condition  $28s_1(M) = 28s_1(M')$  and  $s_i(M) = s_i(M')$  for  $i = 2, 3$ . This follows since the order of the fourth cohomology group of a Wallach space (denoted  $N$  in 4.4 and 5.1) is always odd.

Now we want to explain why  $s_2$  is a homeomorphism invariant for smooth spin manifolds  $M$  of the type under consideration. Let  $W$  be compact topological spin manifold whose boundary is  $M$  such that the class  $u \in H^2(M; \mathbb{Z})$  extends to a class  $z \in H^2(W; \mathbb{Z})$ . Since  $M$  is smooth, the first obstruction for a lift of the topological normal bundle of  $W$  to a linear bundle is an element  $KS(W) \in H^4(W, M; \mathbb{Z}/2)$ . We say that  $W$  is admissible for the computation of  $s_i$  if  $z^2 \cup KS(W) = 0 \in \mathbb{Z}/2$ .

This follows from the fact that both invariants vanish on smooth manifolds and depend only on the class of  $(W, z)$  in the reduced bordism group  $\tilde{\Omega}_8^{\text{TopSpin}}(\mathbb{C}\mathbb{P}^\infty)$  of topological spin manifolds. Moreover, both invariants are non-trivial (cf. [3], Lemma 6.2) and the cokernel of the forgetful homeomorphism  $\tilde{\Omega}_8^{\text{Spin}}(\mathbb{C}\mathbb{P}^\infty) \rightarrow \tilde{\Omega}_8^{\text{TopSpin}}(\mathbb{C}\mathbb{P}^\infty)$  is  $\mathbb{Z}/2$ ; this follows by comparing the Atiyah-Hirzebruch spectral sequences as in [3], Section 6.

As a consequence one can assume, after perhaps adding a closed topological manifold, that for given  $M$  the bounding manifold  $W$  is admissible. Using only admissible  $W$ 's one obtains that  $s_2(M) \bmod \mathbb{Z}$  is also in the spin case a well defined homeomorphism invariant.

The proof of the theorem above for the homeomorphism classification is reduced to the smooth case by noting that if  $\bar{s}_1(M) = \bar{s}_1(M')$  then after adding an appropriate homotopy sphere one can assume that  $s_1(M) = s_1(M')$ .

## References

- [1] L. Astey, E. Micha & G. Pastor, *Homeomorphism and Diffeomorphism types of Eschenburg Spaces*, Differential Geom. Appl. **7** (1997) 41–50.
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- [3] M. Kreck & S. Stolz, *Some nondiffeomorphic homeomorphic homogeneous 7-manifolds with positive sectional curvature*, Differential Geom. **33** (1991) 465–486.

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