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Generalized difference sequence spaces of fuzzy numbers

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Abstract. We introduce new difference sequence spaces of fuzzy numbers: $c_0\left(F, \Lambda, \triangle_n^m, p\right)$, $c\left(F, \Lambda, \triangle_n^m, p\right)$ and $l_\infty\left(F, \Lambda, \triangle_n^m, p\right)$. We then examine some of their topological properties by using a sequence of modulus functions.

Contents

1. Preliminaries, background and notation

A sequence space is defined to be a linear space of real or complex sequences. Throughout the paper $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$ denotes the set of nonnegative integers, the set of real numbers and the set of complex numbers respectively. Let ω denote the space of all sequences (real or complex); l_∞ and c respectively, denotes the space of all bounded sequences, the space of convergent sequences.

Throughout the paper $p = (p_k)$ is a sequence of positive real numbers. The notion of paranormed sequences was studied at the initial stage by Simons [26]. It was further investigated by Maddox [19], Tripathy and Sen [31], Hamid and Neyaz [14], Hamid, Neyaz and Sen [15] and many others.

Following Ruckle [23] and Maddox [19], a modulus function f is a function from $[0, \infty)$ to $[0, \infty)$ such that:

(i) $f(x) = 0$ if and only if $x = 0$.
(ii) $f(x + y) \leq f(x) + f(y)$ $\forall x, y \geq 0$.
(iii) f is increasing.
(iv) f if continuous from the right at $x = 0$.

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The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [32] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming. Matloka [20] introduced bounded and convergent sequences of fuzzy numbers and studied their some properties. Matloka [20] also has shown that every convergent sequence of fuzzy numbers is bounded. Later on sequences of fuzzy numbers have been discussed by Nanda [22], Altin [1], Altinok [2], Başarir and Mursaleen [3], Bilgin [4], Chaudhuri and Das [5], Çolak [6, 7, 8], Diamond and Kloeden [9], Esi [10, 11], Fang and Fang [12], Hamid and Neyaz [13], Hazarika [16], Kelava [17], Savaş [24, 25], Tripathy et al [27, 28, 29, 30], etc.

Let D denote the set of all closed and bounded intervals $X = [a_1, a_2]$ on the real line $\mathbb{R}$. For $X = [a_1, a_2]$ and $Y = [b_1, b_2]$ in D we define

$$d(X : Y) = \max(|a_1 - b_1|, |a_2 - b_2|).$$

It is known that (D, d) is a complete metric space.

Let $I = [0, 1]$. A fuzzy real number X is a fuzzy set on $\mathbb{R}$ and is a mapping $X : \mathbb{R} \to I$ associating each real number t with its grade membership $X(t)$.

A fuzzy real number X is called *convex* if

$$X(t) \geq X(s) \wedge X(r) = \min(X(s), X(s)), \text{ where } s < t < r.$$

A fuzzy real number X is called *normal* if there exists $t_0 \in \mathbb{R}$ such that $X(t_0) = 1$.

A fuzzy real number X is called *upper semi-continuous* if for each $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$ for all $a \in I$ and given $\varepsilon > 0$, $X^{-1}([0, a + \varepsilon))$ is open in the usual topology of $\mathbb{R}$.

The set of all upper semi-continuous, normal, convex fuzzy numbers is denoted by $R(I)$. The α-level set of a fuzzy real number X for $0 < \alpha \leq 1$ denoted by X^α is defined by $X^\alpha = \{t \in \mathbb{R} : X(t) \geq \alpha\}$. The 0-level set is the closure of strong 0-cut.

For each $r \in \mathbb{R}$, $\bar{r} \in \mathbb{R}(I)$ is defined by

$$\bar{r} = \begin{cases} \bar{r}, & \text{if } t = r, \\ 0, & \text{if } t \neq r. \end{cases}$$

The absolute value of $|X|$ of $X \in \mathbb{R}(I)$ is defined by (see for instance Kaleva and Seikkala [17])

$$|X|(t) = \begin{cases} \max\{X(t), X(-t)\}, & \text{if } t \geq 0, \\ 0, & \text{if } t < 0. \end{cases}$$

Let $\bar{d} : \mathbb{R}(I) \times \mathbb{R}(I) \to \mathbb{R}$ be defined by

$$\bar{d}(X, Y) = \sup_{0 \leq \alpha \leq 1} d(X^\alpha, Y^\alpha).$$

Then $\bar{d}$ defines a metric on $\mathbb{R}(I)$ (Matłoka [20]). The additive identity and multiplicative identity in $\mathbb{R}(I)$ are denoted by $\bar{0}$ and $\bar{1}$ respectively.

Throughout the article ω^F, c^F, c_0^F and l_∞^F denote the classes of all, convergent, null, bounded sequence spaces of fuzzy real numbers.

A fuzzy real valued sequence $\{X_n\}$ is said to be convergent to fuzzy real number X, if for $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that $\bar{d}(X,Y) < \varepsilon$ for all $k \geq n_0$.

A fuzzy real valued sequence $\{X_n\}$ is said to be solid (normal) if $(X_k) \in E^F$ implies that $(\alpha_k X_k) \in E^F$ for all sequences of scalars (α_k) with $|\alpha_k| \leq 1$, for all $k \in \mathbb{N}$.

Let $K = \{k_1 < k_2 < \cdots\} \subseteq \mathbb{N}$ and E^F be a sequence space. A k-step space of E^F is a sequence space $\lambda_K^{E^F} = \{(X_{k_n}) \in \omega^F : (X_n) \in E^F\}$.

A canonical preimage of a sequence $\{X_k\} \in \lambda_K^{E^F}$ is a sequence $\{Y_n\} \in \omega^F$ defined as

$$Y_n = \begin{cases} X_n, & \text{if } k \in K, \\ \bar{0}, & \text{otherwise.} \end{cases}$$

A canonical preimage of a step space $\lambda_K^{E^F}$ is a set of all elements in $\lambda_K^{E^F}$, i.e., Y is in canonical preimage of $\lambda_K^{E^F}$ if and only if Y is canonical preimage of some $X \in \lambda_K^{E^F}$.

A sequence space E^F is said to be monotone if it contains the canonical preimages of its step spaces.

A sequence space E^F is said convergence free if $(Y_k) \in E^F$ whenever $(X_k) \in E^F$ and $Y_k = \bar{0}$ whenever $X_k = \bar{0}$.

The difference sequence spaces

$$Z(\Delta) = \{x = (x_k) : \Delta x \in Z\},$$

where $Z = l_\infty$, c and c_0, were studied by Kizmaz [18].

It was further generalized by Tripathy and Esi [28], as follows. Let $m \geq 0$ be an integer. Then

$$H(\Delta^m) = \{x = (x_k) : \Delta^m x \in Z\},$$

for $Z = l_\infty$, c and c_0, where $\Delta^m x_k = x_k - x_{k+m}$. Further, in [27] Tripathy et al. generalized the above notions and unified these as follows:

$$\Delta_n^m x_k = \{\, x \in \omega : (\Delta_n^m x_k) \in Z\},$$

where

$$\Delta_n^m x_k = \sum_{\mu=0}^{n} (-1)^\mu \binom{n}{r} x_{k+m\mu},$$

$$\Delta_n^0 x_k = x_k \forall\ k \in \mathbb{N}.$$

The idea of Kizmaz [18] was applied by Savaş [24, 25] to introduce the notion of difference sequences for fuzzy real numbers and study their different properties. The difference sequence space were further studies by Çolak

[7, 8], Ganie and Sheikh [14], Ganie, Sheikh and Sen [15], Mursaleen [21] and many others.

Let (a_k) and (b_k) be sequences with complex terms and $p = (p_k) \in l_\infty$, we have the following known inequality:

$$|a_k + b_k|^{p_k} \leq B\left(|a_k|^{p_k} + |b_k|^{p_k}\right) \tag{1.1}$$

where $B = \max\{1, 2^{M-1}\}$ and $M = \sup_k p_k$.

2. New classes of sequences of fuzzy numbers

Let $X = (X_k)$ be a sequence of fuzzy numbers and $\Lambda = (f_k)$ be a sequence of moduli. In this article, we define the following classes of difference sequences of fuzzy numbers:

$$c_0\,(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) \,:\, \lim_k [f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k} = 0 \right\},$$

$$c\,(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) \,:\, \lim_k [f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} = 0 \right\},$$

$$l_\infty\,(F, \Lambda, \triangle_n^m, p) = \left\{ X = (X_k) \,:\, \sup_k [f_k(\bar{d}(\triangle_n^m X_k, \bar{0}))]^{p_k} < \infty \right\},$$

for some X_0 and $p = (p_k)$ is a sequence of real numbers such that $p_k > 0$ for all k and $\sup_k p_k = M < \infty$.

Note that for $m = 1 = n$, $f_k(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then these spaces are reduced to $c_0\,(F, \triangle)$, $c\,(F, \triangle)$ and $l_\infty\,(F, \triangle)$, introduced by Mursaleen and Başarir [21]. Again if we take $m = 0$, $n = 1$, $f_k(x) = x$ and $p_k = 1$ for all $k \in \mathbb{N}$, then these spaces are respectively reduced to $c_0\,(F)$, $c\,(F)$ and $l_\infty\,(F)$ introduced by Nanda [22].

Theorem 2.1. *If $\bar{d}$ is a translation invariant metric, then $c_0\,(F, \Lambda, \triangle_n^m, p)$, $c\,(F, \Lambda, \triangle_n^m, p)$ and $l_\infty\,(F, \Lambda, \triangle_n^m, p)$ are closed under the operation of addition and scalar multiplication.*

Proof. As $\bar{d}$ is translation invariant metric, it implies that

$$\bar{d}(\triangle^m X_k + \triangle^m Y_k, X_0 + Y_0) \leq \bar{d}(\triangle_n^m X_k, X_0) + \bar{d}(\triangle_n^m Y_k, Y_0), \tag{2.1}$$

$$\bar{d}(\triangle_n^m \lambda X_k, \lambda X_0) \leq |\lambda| \bar{d}(\triangle_n^m X_k, X_0), \tag{2.2}$$

where λ is a scalar and $|\lambda > 1$. We shall prove only for $c\,(F, \Lambda, \triangle_n^m, p)$. The others can be treated similarly. Suppose that $X = (X_k)$, $Y = (Y_k)$ in $c\,(F, \Lambda, \triangle_n^m, p)$. Then

$$\begin{aligned}
&[f_k(\bar{d}(\triangle_n^m X_k + \triangle_n^m Y_k, X_0 + Y_0))]^{p_k} \\
&\quad \leq [f_k(\bar{d}(\triangle_n^m X_k, X_0) + \bar{d}(\triangle_n^m Y_k, Y_0))]^{p_k}, && \text{by (2.1)} \\
&\quad \leq [f_k(\bar{d}(\triangle_n^m X_k, X_0)) + f_k(\bar{d}(\triangle_n^m Y_k, Y_0))]^{p_k}, && \text{by (ii)} \\
&\quad \leq K^M [f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} + K^M [f_k(\bar{d}(\triangle_n^m Y_k, Y_0))]^{p_k}, && \text{by (1.1).}
\end{aligned}$$

Hence, $X+Y \in c(F,\Lambda,\triangle_n^m,p)$. Let $X=(X_k) \in c(F,\Lambda,\triangle_n^m,p)$. For $\lambda \in \mathbb{R}$ there exists an integer K such that $|\lambda| \leq K$. Then, by taking into account the property (2.2) and the modulus functions f_k for all $k \in \mathbb{N}$, we have

$$\begin{aligned}[f_k(\bar{d}(\lambda\triangle_n^m X_k, \lambda X_0))]^{p_k} &\leq [f_k|\lambda|(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k}, \\ &\leq K^M[f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k}.\end{aligned}$$

This implies that $\lambda X \in c(F,\Lambda,\triangle_n^m,p)$. □

The proof of the following is left to the reader.

Theorem 2.2. *Let $p=(p_k) \in l_\infty$. Then $c_0(F,\Lambda,\triangle_n^m,p)$, $c(F,\Lambda,\triangle_n^m,p)$ and $l_\infty(F,\Lambda,\triangle_n^m,p)$, are paranormed spaces, paranormed by g defined by*

$$g(X)=\sup_k (f(\bar{d}(\triangle_n^m(\alpha_k X_k),\bar{0})))^{\frac{p_k}{M}},$$

where $M=\max(1,\sup_k p_k)$ and $X=(X_k)$.

Theorem 2.3. *Let $\Lambda=(f_k)$ be a sequence of moduli. Then,*

$$c_0(F,\Lambda,\triangle_n^m,p) \subset c(F,\Lambda,\triangle_n^m,p) \subset l_\infty(F,\Lambda,\triangle_n^m,p).$$

Proof. $c_0(F,\Lambda,\triangle_n^m,p) \subset c(F,\Lambda,\triangle_n^m,p)$ is trivial. So, let

$$X=(X_k) \in c(F,\Lambda,\triangle_n^m,p).$$

Then, there is some fuzzy number X_0 such that $\lim_k [f_k(\bar{d}(\triangle_n^m X_k,\bar{0}))]^{p_k}=0$. Now, from (1.1), we have

$$[f_k(\bar{d}(\triangle_n^m X_k,\bar{0}))]^{p_k} \leq K[f_k(\bar{d}(\triangle_n^m X_k, X_0))]^{p_k} + K[f_k(\bar{d}(\triangle_n^m X_k,\bar{0}))]^{p_k}.$$

As $X=(X_k) \in c(F,\Lambda,\triangle_n^m,p)$, we obtain $X=(X_k) \in l_\infty(F,\Lambda,\triangle_n^m,p)$ and this proves the result. □

Theorem 2.4. *The classes $(F,\Lambda,\triangle_n^m,p)$ and $l_\infty(F,\Lambda,\triangle_n^m,p)$ are neither solid nor monotone (in general).*

Proof. Let $f(x)=x$, for all $x \in [0,\infty)$, $m=2,\ n=1,\ \lambda_k=2$ for all $k \in \mathbb{N}$

$$p_k=\begin{cases}1 & \text{for } k \text{ odd},\\ 2 & \text{for } k \text{ even}.\end{cases}$$

Consider the sequence (X_k) defined by $(X_k)=H$ for all $k \in \mathbb{N}$, where

$$H(t)=\begin{cases}t+1, & \text{if } -1 \leq t \leq 0,\\ 1-t, & \text{if } 0 \leq t \leq 1,\\ 0, & \text{otherwise}.\end{cases}$$

Then clearly $(X_k) \in c(F,\Lambda,\triangle_n^m,p)$. For, N, a class of sequences, consider its J-step space N_j defined as follows:

When $(X_k) \in N_j$, then its canonical pre-image $(Y_k) \in N_j$ is given by

$$Y_k = \begin{cases} X_k, & \text{for } k \text{ even}, \\ \bar{0}, & \text{for } k \text{ odd}. \end{cases}$$

Then $(Y_k) \notin c\left(F, \triangle_1^2, p\right)$. Thus, the class of sequences $c\left(F, \triangle_1^2, p\right)$ is not monotone. Therefore, it is not solid. Thus, the class of sequences $c\left(F, \triangle_n^m, p\right)$ is not monotone in general. □

Theorem 2.5. *$c_0\left(F, \Lambda, \triangle_n^m, p\right)$, $c\left(F, \Lambda, \triangle_n^m, p\right)$ and $l_\infty\left(F, \Lambda, \triangle_n^m, p\right)$ are not symmetric in general.*

Proof. We only consider the case $c\left(F, \Lambda, \triangle_n^m, p\right)$. To prove the result we consider the following example:

Let $f(x) = x$, for all $x \in x \in [0, \infty)$, $m = 2$, $n = 1$, $\lambda_k = 3$ and

$$p_k = \begin{cases} 2 & \text{for } K \text{ odd}, \\ 3 & \text{for } k \text{ even}, \end{cases}$$

for all $k \in \mathbb{N}$. Consider the sequence $(X_k) = (H, N, H, N, \dots)$, where the fuzzy number H is defined as follows:

$$H(t) = \begin{cases} t+1, & \text{if } -1 \le t \le 0, \\ 1-t, & \text{if } 0 \le t \le 1, \\ 0, & \text{otherwise}, \end{cases}$$

and the fuzzy number N is defined by

$$N(t) = \begin{cases} \frac{t}{2}+1, & \text{if } -2 \le t \le 0, \\ 1-\frac{t}{2}, & \text{if } 0 \le t \le 2, \\ 0, & \text{otherwise}. \end{cases}$$

Then $(X_k) \in c\left(F, \triangle_1^2, p\right)$. Consider its rearrangement (Y_k) of (X_k) defined by $(Y_k) = (H, N, N, H, H, N, N, H, H, \dots)$. Then $(Y_k) \notin c\left(F, \triangle_1^2, p\right)$. Hence, the class of sequences $c\left(F, \Lambda, \triangle_n^m, p\right)$ is not symmetric, and the result follows. □

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