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Erratum: Behavior of knot invariants under genus 2 mutation

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Abstract. Proposition 2.7 of the original paper (Dunfield, Garoufalidis, Shumakovitch and Thistlethwaite, 2010) is false and as a result Corollary 2.8 has not been established. Here, we provide alternate proofs of the results in our paper which depended on those claims, with the exception of the invariance of generalized knot signatures. In particular, all the results claimed in Table 1.2 of the original paper have still been proved.

The two places where Corollary 2.8 was used are Theorems 2.9 and 3.2. We start by giving a correct proof of Theorem 3.2.

Theorem 3.2. *The colored Jones polynomials of a knot are invariant under $(2, 0)$-mutation for all colors.*

Proof of Theorem 3.2. Let F be a closed genus 2 surface in S^3 disjoint from a knot K, and let K^τ be the mutant of K along F, where here τ is the hyperelliptic involution. We will use that the colored Jones polynomials can be defined via the Kauffman bracket skein module (KBSM), in the style of topological quantum field theory.

The key here is that by Theorem 3.1 of [P], one has the following basis for the KBSM of $F \times I$ where $I = [-1, 1]$: the set of isotopy classes of unoriented links in $F \times \{0\}$ where every component of the link is an essential curve. Here, each such curve is given the blackboard framing. Now the hyperelliptic involution τ acts trivially on this set of framed links and therefore also on KBSM$(F \times I)$.

The surface F divides $S^3 \setminus K$ into two pieces, which we denote by X and Y. Then $S^3 \setminus K^\tau$ is obtained by gluing X to one side of $F \times I$ and Y to the other side via the hyperelliptic involution τ. As τ acts trivially on KBSM$(F \times I)$, it follows that KBSM$(S^3 \setminus K)$ is isomorphic to KBSM$(S^3 \setminus K^\tau)$. By Masbaum

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and Vogel [MV], it follows that the colored Jones polynomials of K and K^τ are equal for all colors. □

We next give a correct proof of part of Theorem 2.9.

Theorem 2.9 (Revised). *The Alexander polynomial of a knot in S^3 does not change under $(2,0)$-mutation.*

The statement of Theorem 2.9 in [DGST] asserts that the generalized signatures are also invariant under $(2,0)$-mutation, but we do not know how to establish this; these signatures are invariant under genus 2 *handlebody* mutation, see [CL].

Proof. The Alexander polynomial of a knot is determined by all of its colored Jones polynomials (this is the Melvin–Morton–Rozansky Conjecture, which was proven in [B-NG]). Thus Theorem 3.2 implies that the Alexander polynomials does not change under $(2,0)$-mutation. □

The problem with Proposition 2.7. Proposition 2.7 claimed that if K is a knot in S^3 which is disjoint from a genus 2 surface F, then either K^τ is obtained from K by various kinds of *handlebody* mutation or $K^\tau \cong K$. In particular, we claimed that if F is incompressible in the complement of K, then in fact F bounds a handlebody in S^3; this is simply false, as the following example shows. Start with a knotted solid torus V in S^3. If we then drill out a tunnel from V, we get a submanifold Y with $F = \partial X$ a genus 2 surface; by choosing a complicated tunnel, we can arrange that F is incompressible in Y. Let X be the complement of Y, and choose a knot K in X which runs through the tunnel and is chosen so that F is incompressible in $X \setminus K$. Then F is incompressible in $S^3 \setminus K$, but it does not bound a handlebody on either side; hence mutation along F is *not* $(2,0)$–handlebody mutation.

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