

ABSTRACT. The paper concerns the topology of an isospectral real smooth manifold for certain Jacobi element associated with real split semisimple Lie algebra. The manifold is identified as a compact, connected completion of the disconnected Cartan subgroup of the corresponding Lie group  $\tilde{G}$  which is a disjoint union of the split Cartan subgroups associated to semisimple portions of Levi factors of all standard parabolic subgroups of  $\tilde{G}$ . The manifold is also related to the compactified level sets of a generalized Toda lattice equation defined on the semisimple Lie algebra, which is diffeomorphic to a toric variety in the flag manifold  $\tilde{G}/B$  with Borel subgroup  $B$  of  $\tilde{G}$ . We then give a cellular decomposition and the associated chain complex of the manifold by introducing colored-signed Dynkin diagrams which parametrize the cells in the decomposition.