

ABSTRACT. The standard Lichnerowicz comparison theorem states that if the Ricci curvature of a closed, Riemannian n -manifold M satisfies $\text{Ric}(X, X) \geq a(n-1)|X|^2$ for every $X \in TM$ for some fixed $a > 0$, then the smallest positive eigenvalue λ of the Laplacian satisfies $\lambda \geq an$. The Obata theorem states that equality occurs if and only if M is isometric to the standard n -sphere of constant sectional curvature a . In this paper, we prove that if M is a closed Riemannian manifold with a Riemannian foliation of codimension q , and if the normal Ricci curvature satisfies $\text{Ric}^\perp(X, X) \geq a(q-1)|X|^2$ for every X in the normal bundle for some fixed $a > 0$, then the smallest eigenvalue λ_B of the basic Laplacian satisfies $\lambda_B \geq aq$. In addition, if equality occurs, then the leaf space is isometric to the space of orbits of a discrete subgroup of $O(q)$ acting on the standard q -sphere of constant sectional curvature a . We also prove a result about bundle-like metrics on foliations: On any Riemannian foliation with bundle-like metric, there exists a bundle-like metric for which the mean curvature is basic and the basic Laplacian for the new metric is the same as that of the original metric.