

ABSTRACT. The three sums named in the title are all known to appear in connection with the complex representation theory of  $GL(2, q)$ . The first two are incarnations of certain spherical vectors, whereas the third is a matrix coefficient for a parabolic basis. In this work, Legendre and Soto-Andrade sums are shown to occur in a second way, as parabolic Clebsch-Gordan coefficients for the tensor product of two Steinberg representations. This realization connects them with Kloosterman sums, and from it we derive a number of identities.