

ABSTRACT. Trudinger and Moser, interested in certain nonlinear problems in differential geometry, showed that if $|\nabla u|^q$ is integrable on a bounded domain in R^n with $q \geq n \geq 2$, then u is exponentially integrable there. Symmetrization reduces the problem to a one-dimensional inequality, which Jodeit extended to $q > 1$. Carleson and Chang proved that this inequality has extremals when $q \geq 2$ is an integer. Hence, so does the Moser-Trudinger inequality (with $q = n$).

This paper extends the result of Carleson and Chang to all real numbers $q > 1$. An application and some related results involving noninteger q are also discussed.