

ABSTRACT. In this paper we study some of the arithmetic structure that is found in a special kind of semi-ring in the isols. These are the semi-rings  $[D(Y), +, \cdot]$  that were introduced by J.C.E. Dekker, and that were later shown by E. Ellentuck to model the true universal recursive statements of arithmetic when  $Y$  is a regressive isol and is hyper-torre (= hereditarily odd-even = HOE). When  $Y$  is regressive and HOE, we further reflect on the structure of  $D(Y)$ . In addition, a new variety of regressive isol is introduced, called combinatorial. When  $Y$  is such an isol, then it is also HOE, and more, and the arithmetic of  $D(Y)$  is shown to have a richer structure.