Yang–Mills theory over surfaces and the Atiyah–Segal theorem

Daniel Ramras

Abstract. Given a representation $f : G \to U(n)$, one can form a vector bundle over the classifying space BG via the mixing construction. When G is a compact Lie group, the Atiyah–Segal theorem starts with this construction and provides a precise relationship between the representation ring $R(G)$ and the topological K-theory of BG. Since the universal cover of a surface is (usually) contractible, one might hope for an analogous theorem relating representations of surface groups to K-theory of surfaces. In this talk, I will describe such a result.

In this setting, the topology of the representation spaces $\mathrm{Hom}(\pi_1 S, U(n))$ becomes crucial. I'll explain how Morse theory for the Yang–Mills functional may be used to study these spaces. In addition to an analogue of the Atiyah–Segal theorem, these considerations provide homotopy-theoretical information about both the representation spaces and the moduli spaces

$$\mathrm{Hom}(\pi_1 S, U(n))/U(n),$$

after stabilizing with respect to rank.