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Erratum to 'Hermitian u-invariants under quadratic field extensions'

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This Erratum fixes a problem in the proof of [1, Prop. 4.3]. For involutions of the first kind (i.e. which restrict to the identity on the center), the proof of [1, Prop. 4.3] covers the case where D is a division algebra, while for unitary involutions, the argument is not correct. Here, we give a different proof for both cases, not assuming D to be a division algebra. However, for involutions of the first kind, we exclude characteristic 2. This does not affect the other results in the article, since we assume everywhere the characteristic of the base field is different from 2 when dealing with involutions of the first kind.

We use the same notation and setup as in [1]. Let K be a field. For a separable field extension L/K with $[L : K] \leqslant 2$, we denote by $\mathsf{can}_{L/K}$ the non-trivial K-automorphism of L if $L \neq K$ and set $\mathsf{can}_{L/K} = \mathsf{id}_K$ otherwise. We recall the following two facts.

Lemma 1. *Let A be a central simple K-algebra and σ an involution on A. Let $F = \{x \in K \mid \sigma(x) = x\}$ and L/F be a separable quadratic field extension contained in A and linearly disjoint from K/F. Then there exists an involution τ on A such that $\tau|_K = \sigma|_K$ and $\tau|_L = \mathsf{id}_L$. If $K = F$, then τ can be choosen to be orthogonal.*

Proof. Consider the involution $\mathsf{can}_{K/F} \otimes \mathsf{id}_L$ on $KL = K \otimes_F L$. We identify KL with the compositum of K and L in A. In particular KL is a simple F-subalgebra of A. It follows by [3, Theorem 4.14] that there exists an involution τ on A such that $\tau|_{KL} = \mathsf{can}_{K/F} \otimes \mathsf{id}_L$ and of the same type as $\tau|_{KL}$, hence satisfying $\tau|_K = \sigma|_K$ as well as $\tau|_L = \mathsf{id}_L$, and either orthogonal or unitary. □

The following lemma is given in [2, Lemma 3.1.1] for any involution in characteristic different from 2. The argument there is valid in arbitrary characteristic. We include an argument for convenience.

Lemma 2. *Let A be a central simple K-algebra, σ an involution on A and $F = \{x \in K \mid \sigma(x) = x\}$. Let L/F be a separable quadratic field extension contained in A which is stable under σ and linearly disjoint from K/F. If $K = F$, assume that $\sigma|_L = \mathsf{id}_L$. Then there exists $j \in A^\times$ such that $\sigma(j) = -j$ and $\mathsf{Int}(j)|_{KL} = \mathsf{can}_{KL/K}$.*

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Proof. Assume first that σ is unitary. Consider the involution $\mathsf{can}_{KL/K} \circ \sigma|_{KL}$ on KL. Note that it extends $\mathsf{can}_{K/F}$. It follows by [3, Theorem 4.14] that there exists an involution τ on A such that $\tau|_K = \mathsf{can}_{K/F}$ and $\tau|_{KL} = \mathsf{can}_{KL/K} \circ \sigma|_{KL}$. By [3, Prop. 2.18], there exist $j' \in A^\times$ such that $\sigma(j') = j'$ and $\tau = \mathsf{Int}(j') \circ \sigma$. Then $\mathsf{Int}(j')|_{KL} = \mathsf{can}_{KL/K}$. Let further $j'' \in K^\times$ be such that $\sigma(j'') = -j''$. Then the element $j = j''j'$ has the claimed properties.

Assume now that $K = F$. By [3, Theorem 4.14], there exists an involution τ on A such that $\tau|_L = \mathsf{can}_{L/F}$. By [3, Prop. 2.7], we have $\tau = \mathsf{Int}(j') \circ \sigma$ for some $j' \in A^\times$ with $\sigma(j') = \varepsilon j'$ where $\varepsilon \in \{\pm 1\}$. Since $\tau|_L = \mathsf{can}_{L/F}$, there exists $j'' \in L^\times$ with $\tau(j'') = -\varepsilon j''$. For $j = j''j'$, we then obtain that $\sigma(j) = -j$ and $\mathsf{Int}(j)|_L = \mathsf{Int}(j')|_L = (\tau \circ \sigma)|_L = \mathsf{can}_{L/F}$, because $\sigma|_L = \mathsf{id}_L$. □

Proposition 3 ([1, Proposition 4.3]). *Let D be a central simple K-algebra and γ an involution on D. Let $F = \{x \in K \mid \gamma(x) = x\}$ and let L/F be a separable quadratic field extension contained in D and linearly disjoint from K/F. Then there exists an involution γ' on D and an element $j \in D^\times$ such that $\gamma'|_K = \gamma|_K$, $\gamma'|_L = \mathsf{can}_{L/F}$, $\mathsf{Int}(j)|_L = \mathsf{can}_{L/F}$ and $\gamma'(j) = -j$. Furthermore, if $K = F$ and $\mathsf{char} K \neq 2$, then γ' can be choosen to be symplectic.*

Proof. In view of Lemma 1, there exists an involution τ on D which is orthogonal or unitary with $\tau|_K = \gamma|_K$ and $\tau|_L = \mathsf{id}_L$. Then, by Lemma 2, there exists some $j \in D^\times$ such that $\tau(j) = -j$ and $\mathsf{Int}(j)|_{KL} = \mathsf{can}_{KL/K}$. The involution $\gamma' = \mathsf{Int}(j) \circ \tau$ satisfies the claims. □

Remark. In the formulation of [1, Prop. 4.3], it is suggested that one can first choose j and then choose γ' accordingly. The formulation of Proposition 3 above claims a simultaneous choice of j and γ' to be possible, which is slightly weaker. The formulation of Proposition 3 is sufficient for its use in proving [1, Prop. 4.5].

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