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# Corrigendum to "Classification of homotopy Dold manifolds", New York J. Math. 9 (2003), 271-293

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The proof of Lemma 6.1, page 286, in the original paper [16], should be changed to the following:

**Lemma 0.1.** *Let* $X = D^m \times P(r,s),\ r,s > 1,\ dim X = n+1$. *If* $x \in L_{n+1}((\mathbb{Z}/2)^{\omega^X})$ *is realized by a normal map* $F : M \to X$ *such that* $F\mid_{\partial M} : \partial M \to \partial X$ *is a CAT homeomorphism, then* $\partial(x) = 0$.

**Proof.** Let $X_1 = D^{m-1} \times P(r,s)$, then $\pi_1(X_1) = \pi_1(X)$, and $\omega^{X_1} = \omega^X$.

Realize the element $-x$ by a normal map $f : N \to X_1 \times I$, such that

$$f\mid_{\partial_- N} : \partial_- N \to X_1 \times 0 \cup \partial X_1 \times I$$

is a CAT homeomorphism (CAT = PL or TOP). By definition of $\partial$, the obstruction to splitting the homotopy CAT structure $f\mid_{\partial_+ N} : \partial_+ N \to X_1 \times 1$ along the submanifold $Y_1 = D^{m-1} \times P(r-1,s)$ is equal to $-\partial x$. Consider the connected sum of the normal maps $F$ and $f$ along the component of boundaries $\partial M$ and $\partial_+ N$ (refer to Browder [2], page 40, 41) to give a normal map

$$F \coprod f : \left(M \coprod N, \partial_- N \cup \partial M \# \partial_+ N\right) \to \left(X \coprod (X_1 \times I), (X_1 \times 0 \cup \partial X_1 \times I) \cup (\partial X \# (X_1 \times 1))\right).$$

According to the construction of surgery obstructions the map $F \coprod f$ is a simple homotopy equivalence, and considered as an element of the group $L_{n+1}((\mathbb{Z}/2)^{\omega^X})$, is equal to zero $(= x + (-x))$; but $\pi_1(X \coprod (X_1 \times I)) \neq \mathbb{Z}/2$. However, by Wall([25]; Th. 9.4), one can change $F \coprod f$ using simultaneous surgeries along 1-cycles in the manifolds $M \coprod N$ and $X \coprod (X_1 \times I)$, without changing the boundaries, which make the fundamental groups equal to $\mathbb{Z}/2$. We obtain as a result of theses surgeries a normal map

$$F_2 : \left(M_2, \partial_- N \cup \partial M \# \partial_+ N\right) \to \left(X_2, (X_1 \times 0 \cup \partial X_1 \times I) \cup (\partial X \# (X_1 \times 1))\right).$$

Since on one component of the boundary the map $F_2$ splits, it follows from a generalization of ([13]; Lemma 1, Section 1.2.2) that $F_2$ splits on the other component of the boundary too. Therefore $\partial(x) = 0$. □

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In the statements 3 (i) and 3(ii) of Theorem (7.6), the factor $\mathbb{Z}/2$ should not be there.

The rest of the paper remains unchanged.

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