New York Journal of Mathematics

New York J. Math. **28** (2022) 1057–1063.

The main supergraph of finite groups

Alireza Khalili Asboei and **Seyed Sadegh Salehi Amiri**

ABSTRACT. Let G be a finite group. The main supergraph $\mathcal{S}(G)$ is a graph with vertex set G in which two vertices x and y are adjacent if and only if $o(x) \mid o(y)$ or $o(y) \mid o(x)$. In this paper, we will show that if $\mathcal{S}(G) \cong \mathcal{S}(S)$, where S belongs to a large class of finite non-solvable groups, then $G \cong S$. This work is an important step in solving Thompson's problem.

CONTENTS

1. Introduction

Let G be a finite group and $x \in G$. The order of x is denoted by $o(x)$. The set of all element orders of G is denoted by $\pi_e(G)$ and the set of all prime factors of $|G|$ is denoted by $\pi(G)$. We set $M_i(G) = |\{g \in G|$ the order of g is $i\}|$. The other notations and terminologies in this paper are standard, and the reader is referred to [14] if necessary.

Define the graph $\mathcal{S}(G)$ with the vertex set G such that two vertices x and y are adjacent if and only if $o(x) \mid o(y)$ or $o(y) \mid o(x)$. This graph is called the main supergraph of $\mathcal{P}(G)$ (power graph of G) and was introduced in [20]. The proper main supergraph $\mathcal{S}^*(G)$ is the graph constructed from $\mathcal{S}(G)$ by removing the identity element of G. We write $x \sim y$ when two vertices x and y are adjacent.

Definition 1.1. *Let G be a finite group. We say that G is recognizable by its main supergraph if for every group H we have $\mathcal{S}(G) \cong \mathcal{S}(H)$, then $G \cong H$.*

Note that not all groups are recognizable by the main supergraph. For example, we have $\mathcal{S}(\mathbb{Z}_4) \cong \mathcal{S}(\mathbb{Z}_2 \times \mathbb{Z}_2)$, but $\mathbb{Z}_4$ is not isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_2$.

Definition 1.2. *Two finite groups G_1 and G_2 are called of the same order type if and only if $M_t(G_1) = M_t(G_2)$ for all t.*

Received August 16, 2021.
2010 *Mathematics Subject Classification.* 20D08, 05C25.
Key words and phrases. same order type groups, non-solvable, main supergraph, Thompson's problem.

ISSN 1076-9803/2022

In 1987, J. G. Thompson [37, Problem 12.37] posed the following problem:
Thompson's Problem. Suppose that G_1 and G_2 are two finite groups of the same order type. If G_1 is solvable, is it true that G_2 is also necessarily solvable?

Another form of this problem can be stated as follows.
Thompson's Problem. Suppose that G_1 and G_2 are two finite groups of the same order type. If G_1 is non-solvable, is it true that G_2 is also necessarily non-solvable?

By definition of the main supergraph, it is clear that if G_1 and G_2 are groups with the same order type, then $\mathcal{S}(G_1) \cong \mathcal{S}(G_2)$. So, if a finite group G is recognizable by the main supergraph, then for G Thompson's problem is true.

In [3], the authors of this paper proved that alternating groups of degree p, $p+1$, $p+2$ and symmetric groups of degree p are recognizable by their main supergraph. Also, in [6], [4], [2] and [5], it is proved that the groups $L_2(p)$, $\mathrm{PGL}_2(p)$, where p is prime, all sporadic simple groups, $L_2(q)$, small Ree groups $^2G_2(3^{2n+1})$, where n is a natural number and Suzuki Ree groups are recognizable by their main supergraph.

The prime graph of G, is denoted by $\Gamma(G)$ and is defined as follows. The vertex set of $\Gamma(G)$ is $\pi(G)$ and two distinct vertices p and q are adjacent if and only if G contains an element of order pq. Let $t(G)$ be the number of connected components of $\Gamma(G)$ and $\pi_i = \pi_i(G)$, $1 \le i \le t(G)$ be the connected components of $\Gamma(G)$. For a group of even order we let $2 \in \pi_1(G)$. Then the order of G can be expressed as the product of $m_1, m_2, \dots, m_{t(G)}$, where m_i $(1 \le i \le t(G))$ are positive integers with $\pi(m_i) = \pi_i$. These m_i $(1 \le i \le t(G))$ are called the order components of G. We write $OC(G) = \{m_1, m_2, \dots, m_{t(G)}\}$ and call it the set of order components of G (see [9]).

In this paper, first we prove if G and S are two arbitrary groups such that $\mathcal{S}(G) \cong \mathcal{S}(S)$, then their order components are equal. Then, we conclude that if S is one of the nonabelian simple groups

- A_p, where p and $p-2$ are primes,
- A_n, where $n = p, p+1, p+2$,
- $L_n(q)$, where $n = 2, 3, 5$,
- $U_p(q)$, $L_{p+1}(q)$,
- $L_{p+1}(2)$,
- $U_{p+1}(q)$,
- $C_n(q)$, where q is an even prime power,
- $C_2(q)$, where $q > 5$,
- $E_8(q)$,
- $F_4(q)$, where $q = 2^n > 2$,
- $G_2(q)$ $(3 \mid q)$,
- $G_2(q)$ $(2 < q \equiv 1 \pmod 3)$,
- $^2G_2(q)$,
- $^3D_4(q)$,
- $^2D_n(3)$, where $9 \le n = 2^m + 1$ not a prime,
- $^2D_{p+1}(q)$, where $5 < p \ne 2^m - 1$,

- ${}^2D_p(3)$, where $p \geq 5$ is a prime number not of the form $2^m + 1$,
- ${}^2D_n(2)$, where $n = 2^m + 1 \geq 5$,
- $D_{p+1}(2)$,
- $D_{p+1}(3)$,
- ${}^2D_n(2)$, where $n = 2^m$,
- Suzuki Ree groups,
- Sporadic simple groups,
- ${}^2E_6(q)$,
- $L_p(q)$,
- $U_n(q)$, where $n = 3, 5, 11$,
- almost sporadic simple groups, except $\mathrm{Aut}(McL)$ and $\mathrm{Aut}(J_2)$,
- S_n, for $n = p, p+1$, where $p \geq 3$ is a prime number,

then S is recognizable by the main supergraph.

Main Theorem. Let G and S be two arbitrary groups such that $\mathcal{S}(G) \cong \mathcal{S}(S)$. Then $OC(G) = OC(S)$.

To get the main result of this paper, we need to the following lemma.

Lemma 1.3. *Let $OC(G) = OC(S)$, where S is one of the nonabelian simple groups listed before the main theorem, then $G \cong S$.*

Proof. See [1, 7, 8, 10, 11, 12, 13, 17, 15, 18, 16, 21, 24, 25, 26, 27, 23, 22, 28, 29, 31, 32, 36, 34, 35, 33, 30, 38, 39]. □

Corollary 1.4. *Let G be a finite group listed in the above lemma. Then G is recognizable by the main supergraph.*

Corollary 1.5. *Let G be a finite group listed in the above lemma. Then Thompson's problem is true for G.*

According to research conducted on non-solvable groups of order less than 2000 by using GAP and Corollary 1.4, we pose the following two conjectures:

Conjecture 1.6. *Let S be a finite simple group and G be an arbitrary finite group such that $\mathcal{S}(G) \cong \mathcal{S}(S)$. Then $G \cong S$.*

Conjecture 1.7. *Let S be a finite non-solvable group and G be an arbitrary finite group such that $\mathcal{S}(G) \cong \mathcal{S}(S)$. Then G is a non-solvable group.*

It is clear that if Conjecture 1.7 is true, then Thompson's problem is also true. Note that there exist non-solvable groups S and G such that $\mathcal{S}(G) \cong \mathcal{S}(S)$, but G and S are not isomorphic. For example, if $G = \mathbb{Z}_4 \times A_5$ and $S = < a, b >$, where

$$a = (1, 20, 17, 5, 12)(2, 3, 9, 19, 10)(4, 14, 22, 11, 6)(7, 8, 15, 13, 16)$$

$$b = (2, 18)(5, 11)(6, 21)(7, 24)(9, 17)(10, 16)(12, 23)(13, 20)(14, 19)(15, 22)$$

(in fact S has structure description $\mathrm{SL}(2, 5) : \mathbb{Z}_2$), then $\mathcal{S}(G) \cong \mathcal{S}(S)$, but G and S are not isomorphic.

The next lemma is used in the proof of the main theorem.

Lemma 1.8. [40, Theorem 3] *Let G be a finite group. Then the number of elements whose order is a multiple of n is either zero, or a multiple of the greatest divisor of $|G|$ that is prime to n.*

2. Proof of main theorem

By definition of the main supergraph and our assumption, we have $|G| = |S|$ and $\mathcal{S}^*(S) \cong \mathcal{S}^*(G)$. First, let $\mathcal{S}^*(G)$ be a connected graph. We show that $\Gamma(G)$ and $\Gamma(S)$ are connected. If $\pi(G) = \{p\}$, then $\Gamma(G) = \Gamma(S)$ has one vertex. So, $\Gamma(G)$ and $\Gamma(S)$ are connected. Let $|\pi(G)| \geq 2$. If $\Gamma(G)$ or $\Gamma(S)$ are disconnected, then $\Gamma(G)$ or $\Gamma(S)$ have two or more connected components. By definition of the main supergraph $\mathcal{S}^*(G)$ is a disconnected graph, which is a contradiction.

Now, let $\mathcal{S}^*(G)$ be a disconnected graph. Then it has two or more connected components. Suppose that K_1 and K_2 are two connected components of $\mathcal{S}^*(G)$. We show that if x, y are two arbitrary vertices of K_1 and K_2, respectively such that $o(x) = r$ and $o(y) = s$, where r and s are primes, then r and s are not joined by an edge in the prime graph of G. Assume that r and s are joined by an edge in the prime graph of G. Then $rs \in \pi_e(G)$. So, there exists an element of order rs in G. Assume $z \in G$ and $o(z) = rs$. By definition of the main supergraph $x \sim z$ and $y \sim z$. Thus K_1 and K_2 are connected, which is a contradiction.

Suppose that K_1, K_2,..., and K_n are all connected components of $\mathcal{S}^*(G)$. Let $\pi(K)$ be all prime numbers that divide the order of vertices of K, where K is one of the connected components of $\mathcal{S}^*(G)$. We claim that $\pi(K_i)$ for every $1 \leq i \leq n$ is the set of vertices of one of the connected components of $\Gamma(G)$. Assume that T_i is a component in the prime graph G such that the vertices of T_i are subset of $\pi(K_i)$. Thus, $rs \notin \pi_e(G)$ for every $r \in V(T_i)$ and $s \in \pi(K_i)\setminus V(T_i)$. By definition of the main supergraph, we can conclude that K_i is not connected, a contradiction. Therefore, there exists a one-to-one correspondence between connected components of $\mathcal{S}^*(G)$ and $\Gamma(G)$. It follows that $\pi(K_i)$ for every $1 \leq i \leq n$ is the set of vertices of one connected component of $\Gamma(G)$.

Let K be one of the connected components of $\mathcal{S}^*(G)$. The vertices of K are elements of G and their orders are divided by some prime numbers. We will show how to find these prime numbers.

Suppose that K_1, K_2 are two arbitrary connected components of $\mathcal{S}^*(G)$. Since K_1 and K_2 are isolated, we have $rs \notin \pi_e(G)$, where $r \in \pi(K_1)$ and $s \in \pi(K_2)$.

Let $p \in \pi(K_1)$ be arbitrary. If $\pi(K_2) = \{p_1\}$, then considering $n = p_1$ in Lemma 1.8, $|P| \mid \sum_{t \text{ is a multiple of } p_1} M_t = (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k}) = |K_2|$ ($p_1^k \in \pi_e(G)$), where P is a Sylow p-subgroup of G. Assume that $\pi(K_2) = \{p_1, p_2\}$. Considering $n = p_1p_2$ in Lemma 1.8, we have $|P| \mid \sum_{t \text{ is a multiple of } p_1p_2} M_t$. On the other hand, considering $n = p_1$ in Lemma 1.8, $|P| \mid \sum_{t \text{ is a multiple of } p_1} M_t = (\sum_{t \text{ is a multiple of } p_1p_2} M_t) + (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k})$. It follows that $|P| \mid (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k})$. Similarly, $|P| \mid (M_{p_2} + M_{p_2{}^2} + \cdots + M_{p_2^e})$ ($p_2^e \in \pi_e(G)$). Therefore, $|P| \mid (M_{p_1} + M_{p_1{}^2} + \cdots + M_{p_1^k}) + (M_{p_2} + M_{p_2{}^2} + \cdots + M_{p_2^e}) + (\sum_{t \text{ is a multiple of } p_1p_2} M_t) = |K_2|$. If $\pi(K_2) = \{p_1, p_2, p_3\}$, then considering $n = p_1p_2p_3$, p_1p_2, p_1p_3 and p_1

in Lemma 1.8, $|P| \mid \sum_{t \text{ is a multiple of } p_1p_2p_3} M_t$, $|P| \mid \sum_{t \text{ is a multiple of } p_1p_2} M_t$, $|P| \mid \sum_{t \text{ is a multiple of } p_1p_3} M_t$ and $|P| \mid \sum_{t \text{ is a multiple of } p_1} M_t$. Thus, $|P| \mid (M_{p_1}+M_{p_1^2}+\cdots+M_{p_1^k})$. Similarly, $|P| \mid (M_{p_2}+M_{p_2^2}+\cdots+M_{p_2^e})$ and $|P| \mid (M_{p_3}+M_{p_3^2}+\cdots+M_{p_3^l})$. Therefore, $|P| \mid |K_2|$.

Arguing as above, if $\pi(K_2) = \{p_1, p_2, .., p_r\}$, then $|P| \mid |K_2|$. Also, $|P| \mid |K_i|$ for every connected component K_i $(i \neq 1)$ of $\mathcal{S}^*(G)$. Since $|P| \mid |G| = (1 + |K_1| + |K_2| + \cdots + |K_n|)$, we have $|P| \nmid |K_1|$. Now, let K be one of the connected components of $\mathcal{S}^*(G)$. If $p \in \pi(G)$ is such that $|P| \nmid |K|$, then $p \in \pi(K)$.

Since there exists a one-to-one correspondence between connected components of $\mathcal{S}^*(G)$ and $\Gamma(G)$ and $|G| = |S|$, we have $OC(G) = OC(S)$. This completes the proof of the main theorem.

3. Acknowledgment

We thank the referee for some extremely helpful remarks, which allowed us to improve the paper.

References

[1] Alavi, Seyed H.; Daneshkhah, Ashraf. A new characterization of alternating and symmetric groups. *J. Appl. Math. Comput.* **17** (2005), no. 1-2, 245–258. MR2108803, Zbl 1066.20012, doi: 10.1007/BF02936052. 1059

[2] Amiri, Seyed Sadegh Salehi; Asboei, Alireza K. Recognition of $L_2(q)$ by the main supergraph, *Iran. J. Math. Sci. Inform.* **16** (2021), no. 1, 137–144. MR4259531, Zbl 1477.20030. 1058

[3] Asboei, Alireza Khalili; Amiri, Seyed Sadegh Salehi. Some alternating and symmetric groups and related graphs. *Beitr Algebra Geom.* **59** (2018), no. 1, 21–24. MR3761396, Zbl 1387.20007, doi: 10.1007/s13366-017-0360-8. 1058

[4] Asboei, Alireza Khalili; Amiri, Seyed Sadegh Salehi. The small Ree group ${}^2G_2(3^{2n+1})$ and related graph. *Comment. Math. Univ. Carolin*. **59** (2018), no. 3, 271–276. MR3861551, Zbl 1463.20012, doi: 10.14712/1213-7243.2015.255. 1058

[5] Asboei, Alireza Khalili; Amiri, Seyed Sadegh Salehi. Recognizability of finite groups by Suzuki group. *Arch. Math. (Brno)* **55** (2019), no. 4, 225–228. MR4038357, Zbl 1463.20013, doi: 10.5817/AM2019-4-225. 1058

[6] Asboei, Alireza Khalili; Amiri, Seyed Sadegh Salehi. Some results on the main supergraph of finite groups. *Algebra Discrete Math*, **30** (2020), no. 2, 172–178. MR4210243, Zbl 07414236, doi: 10.12958/adm584. 1058

[7] Chen, Guiyun Y. A new characterization of sporadic simple groups. *Algebra Colloq.* **3** (1996), no. 1, 49–58. MR1374160, Zbl 0845.20011. 1059

[8] Chen, Guiyun Y. A new characterization of $E_8(q)$. *J. Southwest China Normal Univ.* **21** (1996), no. 3, 215–217. 1059

[9] Chen, Guiyun Y. On Thompson's conjecture. *J. Algebra* **185** (1996), no. 1, 184–193. MR1409982, Zbl 0861.20018, doi: 10.1006/jabr.1996.0320. 1058

[10] Chen, Guiyun Y. A new characterization of Suzuki–Ree groups. *Sci. China (Ser A).* **40** (1997), no. 8, 807–812. MR1481680, Zbl 0890.20015, doi: 10.1007/BF02878919. 1059

[11] Chen, Guiyun Y. A new characterization of $PSL_2(q)$. *Southeast Asian Bull. Math.* **22** (1998), no. 3, 257–263. MR1684163, Zbl 0936.20010. 1059

[12] Chen, Guiyun Y. Characterization of ${}^3D_4(q)$. *Southeast Asian Bull. Math.* **25** (2001), no. 3, 389–401. MR1933944, Zbl 1007.20015. 1059

[13] Chen, Guiyun Y.; Shi, Huaguo. $^2D_n(3)(9 \leq n = 2^m + 1$ not a prime) can be characterized by its order components. *J. Appl. Math. Comput.* **19** (2005), no 1-2, 353–362. MR2162329, Zbl 1078.20016, doi: 10.1007/BF02935810. 1059

[14] Conway, John H.; Curtis, Robert T.; Norton, Simon P.; Parker, Richard A.; Wilson, Robert A. Atlas of finite groups. Maximal subgroups and ordinary characters for simple groups. *Oxford University Press, Eynsham*, 1985. xxxiv+252 pp. ISBN: 0-19-853199-0. MR0827219, Zbl 0568.20001. 1057

[15] Darafsheh, Mohammad R. Characterizability of the group $^2D_p(3)$ by its order components, where $p \geq 5$ is a prime number not of the form $2^m + 1$. *Acta Math. Sin., (Engl. Ser).* **24** (2008), no. 7, 1117–1126. MR2420882, Zbl 1157.20005, doi: 10.1007/s10114-007-6143-7. 1059

[16] Darafsheh, Mohammad R. Characterization of the groups $D_{p+1}(2)$ and $D_{p+1}(3)$ using order components. *J. Korean Math.Soc.* **47** (2010), no. 2, 311–329. MR2605984, Zbl 1198.20014, doi: 10.4134/JKMS.2010.47.2.311. 1059

[17] Darafsheh, Mohammad R., Mahmiani, Anehgaldi. A quantitative characterization of the linear groups $L_{p+1}(2)$ where p is a prime number. *Kumamoto J. Math.* **20** (2007), 33–50. MR2316615, Zbl 1136.20009. 1059

[18] Darafsheh, Mohammad R., Mahmiani, Anehgaldi. A characterization of the group $^2D_n(2)$, where $n = 2^m + 1 \geq 5$. *J. Appl. Math. Comput.* **31** (2009), no. 1-2, 447–457. MR2545741, Zbl 1185.20015, doi: 10.1007/s12190-008-0223-4. 1059

[19] The GAP Group. GAP – Groups, Algorithms, and Programming, Version 4.11.1, 2021. https://www.gap-system.org

[20] Hamzeh, Asma; Ashrafi, Ali R. Automorphism groups of supergraphs of the power graph of a finite group. *European J. Combin.* **60** (2017), 82–88. MR3567537, Zbl 1348.05095, doi: 10.1016/j.ejc.2016.09.005. 1057

[21] Iranmanesh, Ali; Alavi, Seyed H. A characterisation of simple groups $PSL(5, q)$ *Bull. Austral. Math. Soc.* **65** (2002), no. 2, 211–222. MR1898535, Zbl 1005.20015, doi: 10.1017/S0004972700020256. 1059

[22] Iranmanesh, Ali; Alavi, Seyed H.; Khosravi, Behrooz. A characterization of $PSL(3, q)$ for $q = 2^m$. *Acta Math. Sin.* (Engl. ser.) **18** (2002), no. 3, 463–472. MR1929221, Zbl 1016.20005, doi: 10.1007/s101140200164. 1059

[23] Iranmanesh, Ali; Alavi, Seyed H.; Khosravi, Behrooz. A characterization of $PSL(3, q)$ where q is an odd prime power. *J. Pure Appl. Algebra.* **170** (2002), no. 2-3, 243–254. MR1904845, Zbl 1001.20005, doi: 10.1016/S0022-4049(01)00113-X. 1059

[24] Iranmanesh, Ali; Khosravi, Behrooz. A characterization of $F_4(q)$ where $q = 2^n$ $(n > 1)$. *Far East J. Math. Sci. (FJMS)* **2** (2000), no. 6, 853–859. MR1808700, Zbl 0972.20010. 1059

[25] Iranmanesh, Ali; Khosravi, Behrooz. A characterization of $C_2(q)$ where $q > 5$. *Comment. Math. Univ. Carolin.* **43** (2002), no. 1, 9–21. MR1903303, Zbl 1068.20020. 1059

[26] Iranmanesh, Ali; Khosravi, Behrooz. A characterization of $PSU_5(q)$. *Int. Math. J.* **3** (2003), no. 2, 129–141. MR1952515, Zbl 1158.20304. 1059

[27] Iranmanesh, Ali; Khosravi, Behrooz. A characterization of $PSU_{11}(q)$. *Canad. Math. Bull.* **47** (2004), no. 4, 530–539. MR2099751, Zbl 1066.20016, doi: 10.4153/CMB-2004-052-8. 1059

[28] Iranmanesh, Ali; Khosravi, Behrooz; Alavi, Seyed H. A characterization of $PSU_3(q)$ for $q > 5$. *Southeast Asian Bull. Math.* **26** (2002), no. 1, 33–44. MR2046570, Zbl 1019.20012, doi: 10.1007/s100120200024. 1059

[29] Khosravi, Amir; Khosravi, Behrooz. A new characterization of almost sporadic groups. *J. Algebra Appl.* **1** (2002), no. 3, 267–279. MR1932151, Zbl 1033.20013, doi: 10.1142/S021949880200015X. 1059

[30] Khosravi, Amir; Khosravi, Behrooz. A characterization of $^2D_n(q)$, where $n = 2^m$. *Int. J. Math. Game Theory Algebra*. **13** (2003), no. 3, 253–265. MR2044947, Zbl 1071.20017. 1059

[31] KHOSRAVI, AMIR; KHOSRAVI, BEHROOZ. A new characterization of $PSL(p,q)$. *Comm. Algebra.* **32** (2004), no. 6, 2325–2339. MR2100473, Zbl 1071.20018, doi: 10.1081/AGB-120037223. 1059

[32] KHOSRAVI, AMIR; KHOSRAVI, BEHROOZ. r-Recognizability of $B_n(q)$ and $C_n(q)$ where $n = 2^m \geq 4$. *J. Pure Appl. Algebra.* **199** (2005), no. 1-3, 149–165. MR2134298, Zbl 1076.20008, doi: 10.1016/j.jpaa.2004.11.010. 1059

[33] KHOSRAVI, AMIR; KHOSRAVI, BEHROOZ. Charactenzability of $PSU(p+1,q)$ by its order component(s). *Rocky Mountain J. Math*. **36** (2006), no. 5, 1555–1575. MR2285300, Zbl 1138.20013, doi: 10.1216/rmjm/1181069382. 1059

[34] KHOSRAVI, BEHROOZ; KHOSRAVI, BAHMAN. A characterization of ${}^2E_6(q)$. *Kumamoto J. Math.* **16** (2003), 1–11. MR1975291, Zbl 1046.20015. 1059

[35] KHOSRAVI, BEHROOZ; KHOSRAVI, BAHMAN; KHOSRAVI, BEHNAM. Characterizability of $PSL(p+1,q)$ by its order component(s). *Houston J. Math.* **32** (2006), no. 3, 683–700. MR2247903, Zbl 1108.20010. 1059

[36] KHOSRAVI, BEHROOZ; KHOSRAVI, BEHNAM. A characterization of $E_6(q)$. *Algebras, Groups Geom.* **19** (2002), no. 2, 225–243. MR1906493, Zbl 1167.20310. 1059

[37] KHUKHRO, EVGENY I.; MAZUROV, VICTOR D. Unsolved problems in group theory. The Kourovka Notebook. No. 18 (English version). Preprint, 2015. arXiv:1401.0300v6. 1058

[38] NOSRATPOUR P. Characterization of $G_2(q)$, where $2 < q \equiv 1(mod\ 3)$ by order components. *J. Linear. Topol. Algebra.* **4** (2015), no. 1, 11–23. MR3564598, Zbl 1412.20007. 1059

[39] SHI, HUAGUO; CHEN, GUIYUN Y. ${}^2D_{p+1}(2)(5 \leq p \neq 2^m - 1)$ can be characterized by its order components. *Kumamoto J. Math.* **18** (2005), 1-8. MR2144892, Zbl 1077.20027. 1059

[40] WEISNER, LOUIS. On the number of elements of a group which have a power in a given conjugate set. *Bull. Amer. Math. Soc.* **31** (1925), no. 9-10, 492–496. MR1561103, JFM 51.0112.05, doi: 10.1090/S0002-9904-1925-04087-2.

1060

(Alireza Khalili Asboei) DEPARTMENT OF MATHEMATICS, FARHANGIAN UNIVERSITY, TEHRAN, IRAN
a.khalili@cfu.ac.ir

(Seyed Sadegh Salehi Amiri) DEPARTMENT OF MATHEMATICS, BABOL BRANCH, ISLAMIC AZAD UNIVERSITY, BABOL, IRAN
salehisss@baboliau.ac.ir

This paper is available via http://nyjm.albany.edu/j/2022/28-43.html.