

ABSTRACT. Given a stationary process $(X_p)_{p \in \mathbb{Z}}$ and an event $B \in \sigma(X_p, p \in \mathbb{Z})$, we study the almost sure convergence as n and m go to infinity of the “bilateral” martingale

$$\mathbb{E}[\mathbf{1}_B \mid X_{-n}, X_{-n+1}, \dots, X_{m-1}, X_m].$$

We show that almost sure convergence holds in some classical examples such as i.i.d. or Markov processes, as well as for the natural generator of Chacon’s transformation. However, we also prove that in every aperiodic dynamical system with finite entropy, there exists a generating process and a measurable set B for which the almost sure convergence of the bilateral martingale does not hold.