

ABSTRACT. Let  $L$  be a Galois extension of  $K$ , finite field extensions of  $\mathbb{Q}_p$ ,  $p$  odd, with Galois group cyclic of order  $p^2$ . There are  $p$  distinct  $K$ -Hopf algebras  $A_d$ ,  $d = 0, \dots, p - 1$ , which act on  $L$  and make  $L$  into a Hopf Galois extension of  $K$ . We describe these actions. Let  $R$  be the valuation ring of  $K$ . We describe a collection of  $R$ -Hopf orders  $E_v$  in  $A_d$ , and find criteria on  $E_v$  for  $E_v$  to be the associated order in  $A_d$  of the valuation ring  $S$  of some  $L$ . We find criteria on an extension  $L/K$  for  $S$  to be  $E_v$ -Hopf Galois over  $R$  for some  $E_v$ , and show that if  $S$  is  $E_v$ -Hopf Galois over  $R$  for some  $E_v$ , then the associated order  $\mathcal{A}_d$  of  $S$  in  $A_d$  is Hopf, and hence  $S$  is  $\mathcal{A}_d$ -free, for all  $d$ . Finally we parametrize the extensions  $L/K$  whose ramification numbers are  $\equiv -1 \pmod{p^2}$  and determine the density of the parameters of those  $L/K$  for which the associated order of  $S$  in  $KG$  is Hopf.