Abstract. Let M be a noncompact manifold and let $\Gamma_c^\infty(S^2(M))$ (respectively $\Gamma_c^\infty(T^1(M))$) be the LF space of 2-covariant symmetric tensor fields (resp. 1-forms) on M, with compact support. Given any Riemannian metric g on M, the first-order differential operator $\delta^* : \Gamma_c^\infty(T^1(M)) \to \Gamma_c^\infty(S^2(M))$ can be defined by $\delta^*\omega = 2\,\mathrm{symm}\,\nabla\omega$, where ∇ denotes the Levi-Civita connection of g.

The aim of this paper is to prove that the subspace $\mathrm{Im}\,\delta^*$ is closed and to show several examples of Riemannian manifolds for which $\Gamma_c^\infty(S^2(M)) \neq \mathrm{Im}\,\delta^* \oplus (\mathrm{Im}\,\delta^*)^\perp$, where orthogonal is taken with respect to the usual inner product defined by the metric.